PROPAGATION CHARACTERISTICS OF THE RIDGE WAVEGUIDE FOR ACOUSTIC SURFACE WAVES*

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Abstract

Using a microwave network approach, approximate but simple dispersion relations are obtained for the "flexural" and "pseudo-Rayleigh" modes of the ridge waveguide. The propagation characteristics calculated from the relation for the "flexural" mode compare very well with experimental data over a wide frequency range; no accurate measurements are presently available for the "pseudo-Rayleigh" mode.

Summary

There is, at the present time, considerable interest in the ridge waveguide because of the recent experimental observation of a very slow "flexural" mode,[1] so called because of its close relationship with the flexural mode of an infinite plate. Such a slow mode is necessarily characterized by strong confinement of the fields to the guiding (ridge) region, under which conditions it is possible to realize a wide variety of waveguide components analogous to their electromagnetic counterparts.[2] In addition to this flexural mode, which is highly dispersive, the ridge guide can also support an essentially nondispersive "pseudo-Rayleigh" mode,[1] so called because of its similarity with the Rayleigh wave on a free surface. Although not as slow as the flexural mode, the pseudo-Rayleigh mode should also carry most of its energy in the ridge region, for reasons to be discussed later, and has the virtue of being essentially dispersionless.

In the present paper, we employ a microwave network approach in an approximate analysis of the propagation characteristics of the pseudo-Rayleigh and flexural modes (see Fig. 1), which are in fact the lowest even and odd modes, respectively, of the ridge waveguide. From a transverse equivalent network for each mode, one obtains approximate but simple analytic dispersion relations, from which the propagation characteristics are easily computed. Comparison of the latter with available experimental data[1] shows very good agreement over a wide range of frequency.

In the formulation of the problem, the cross section of the ridge guide is recognized to consist of a plate of length H, terminated at one end in a free edge, and joined at the other end to a semi-infinite substrate of the same material. If the width 2W of the ridge is sufficiently small, it is natural and convenient to represent the fields in the ridge (plate) region in terms of plate modes, since the number of such modes propagating would then be small, and the boundary conditions on the sides of the ridge are automatically satisfied. Each propagating mode (with real wavenumber) is then rigorously represented in terms of an equivalent transmission line in the z direction.[3] Modes with imaginary or complex wavenumber manifest themselves as energy stored in the vicinity of the free edge and ridge-substrate junctions and, in an exact representation, are best accounted for by including their effect in the lumped equivalent networks representing these junctions. In the substrate region, the bulk waves are the natural set of modes to employ in the representation, and these must be coupled to the plate modes of the ridge region in a manner that satisfies the continuity conditions for velocity and stress at the ridge-substrate junction, and also the stress-free condition at the substrate surface. In the present analysis, rigorous representations are employed for the propagating plate modes in the ridge region and for the principal bulk-wave contributions in the substrate, but the representations for the junction discontinuities are approximate.

Figure 2 shows the approximate transverse equivalent network for odd modes of the ridge waveguide, valid for the frequency range $2W < \lambda_s/2$. In this range, the lowest odd Lamb mode of a plate is the only propagating mode in the ridge region, and this mode is represented by the transmission line[3] with wavenumber κ_f and characteristic impedance Z_f, where the definitions of these quantities are found in Fig. 2. Since the particle motion associated with this mode is primarily in the x direction,[1] the dominant contribution to the fields in the substrate will consist of SH bulk waves with particle motion in the x direction. Of these waves, it can be shown that the constituent with no x variation is the most important, and this SH wave is then represented by the transmission line[3] with parameters κ_s and Z_s, as defined in Fig. 2. The approximate representation of the free edge of the ridge as a simple open circuit, and a similar representation of the ridge-substrate junction by an equally simple transformer, result from prior insight into the physical behavior of the mode, which greatly facilitates the proper choice of approximations. In these approximations, one neglects the higher plate modes and other bulk waves which are excited at the junctions and stored in the vicinity thereof.

The (approximate) dispersion relation for the flexural mode of the ridge guide can now be written down by inspection of the equivalent network of Fig. 2, upon use of the transverse resonance condition

$$\overleftarrow{Z}(T) + \overrightarrow{Z}(T) = 0 \qquad (1)$$

where $\overleftrightarrow{Z}(T)$ are the impedances seen looking in opposite directions at any reference plane T. After some elementary simplification, Eq. (1) leads to the dispersion relation

$$\sqrt{k_f^2 - k_y^2} = \frac{k_f^2}{k_s^2}\sqrt{k_y^2 - k_s^2}\,\cot\sqrt{k_f^2 - k_y^2}\,H \quad , \qquad (2)$$

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which is valid, within the frequency range $2W < \lambda_s/2$, for the flexural mode and all odd higher modes of the ridge guide. The solutions of the dispersion relation that are of interest are such that $k_f > k_y > k_s$, so that κ_f is real and $\kappa_s = -j|\kappa_s|$. Under these conditions, the transverse (z) variation of the modal field is trigonometric in the ridge region, but exponentially decaying into the substrate, so that most of the energy resides within the ridge and is distributed throughout its height in a standing wave fashion.

A numerical solution of Eq. (2) yields the propagation characteristics of the flexural mode of the ridge guide, as shown in Fig. 3, where the phase velocity $v_p = \omega/k_y$, normalized to the Rayleigh wave velocity v_R, is plotted as a function of $k_R(2W) = \omega 2W/v_R$. Agreement with the experimental results of Mason et al.,[1] is seen to be very good for a ridge with 2:1 aspect ratio, and to be excellent for the 3:1 case. The 1:1 case is not as good, but nevertheless still yields a rather accurate prediction of the velocity minimum, if not its location in frequency.

Figure 4 shows the transverse equivalent network for even modes of the ridge guide, which is valid for the frequency range $2W < \lambda_s/3$. In this case, the fields in the ridge are represented in terms of the lowest even Lamb mode and lowest SH mode of a plate, and the coupling of these modes at the top of the ridge is represented approximately by the transformer network shown. Since the particle motion for these modes is primarily in the yz plane, it can be shown that the dominant contribution to the substrate fields will consist of a bulk P wave and a bulk SV wave, each having no variation in x. The coupling between these bulk waves and the plate waves is approximated by a direct connection. It is noted that the two "S" lines are in fact identical, but that the "P" and "L" lines are different.

The application of the transverse resonance condition, (1), leads to the dispersion relation for even modes of the ridge guide:

$$(k_s^2 - 2k_y^2)^2 \frac{\sqrt{k_y^2 - k_L^2} + \sqrt{k_y^2 - k_p^2}\tanh\sqrt{k_y^2 - k_L^2}\,H}{\sqrt{k_y^2 - k_p^2} + \sqrt{k_y^2 - k_L^2}\tanh\sqrt{k_y^2 - k_L^2}\,H} = 4k_s^2\sqrt{k_y^2 - k_s^2}\sqrt{k_y^2 - k_L^2} \qquad (3)$$

In the type of solutions sought, $k_y > k_s > k_L > k_p$, so that κ_p, κ_L and κ_s are all imaginary, and the fields actually decay down from the top of the ridge in a certain fashion, and then continue to decay into the substrate in a slightly different manner. From a numerical solution of (3), one obtains the propagation characteristics of the lowest mode of even symmetry, the pseudo-Rayleigh mode, as shown in Fig. 5. For this mode, the propagation behavior is almost independent of aspect ratio, and the dispersion curves for H/2W = 1, 2 and 3 are indistinguishable on the scale of Fig. 5. The reason for this is found in the fact that the fields decay down from the top of the ridge so that the effect of the ridge-substrate junction is almost equally small for all ridge heights.

Since the phase velocity is only slightly lower than the Rayleigh velocity, the lateral decay rate will be small, but the magnitude of the fields at the bottom of the ridge are small to begin with because of the decay from the top of the ridge, so that the energy is still concentrated within the ridge region. In Fig. 5, the phase velocity is seen to be essentially constant over a wide frequency range. In nondispersive applications, therefore, the use of this pseudo-Rayleigh mode is to be preferred over the flexural mode.

In conclusion, it is noted that a judicial choice of approximations, based upon physical insight, has led to very simple solutions, of good accuracy over a wide range of frequencies.

References

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2. R.A. Waldron, "Microsound Waveguides and Waveguide Components," IEEE Trans. SU-18, 219-230 (1971).

3. A.A. Oliner, R.C.M. Li and H.L. Bertoni, "Microwave Network Approach to Guided Acoustic Surface Wave Structures," Final Report to U.S. Army Electronics Command, ECOM-0418-F, Polytechnic Institute of Brooklyn, Report No. PIBEP-71-092, Farmingdale, N.Y.; August 1971.

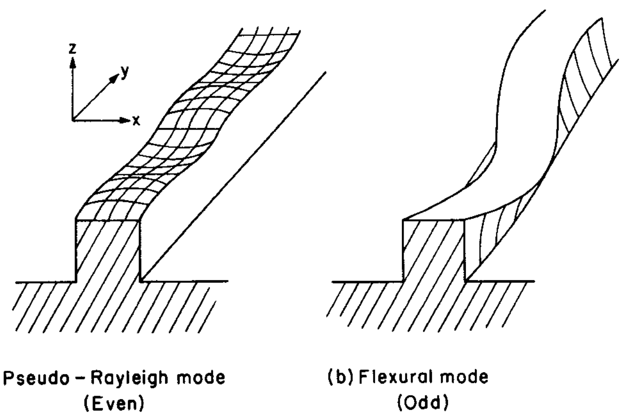
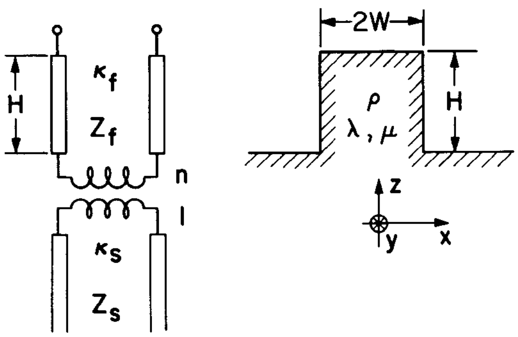


FIG. 1 - LOWEST MODES OF RIDGE WAVEGUIDE (AFTER MASON ET AL.[1]).



$$\kappa_s = \sqrt{k_s^2 - k_y^2} \quad , \quad \kappa_f = \sqrt{k_f^2 - k_y^2}$$

$$Z_s = \frac{\omega \rho}{\kappa_s} \quad , \quad Z_f = \frac{\omega \rho}{\kappa_f}$$

$$n = \frac{k_s}{k_f} \quad , \quad k_s = \omega \sqrt{\frac{\rho}{\mu}}$$

k_f = wavenumber of lowest flexural Lamb wave

FIG. 2 - TRANSVERSE EQUIVALENT NETWORK FOR ODD MODES OF THE RIDGE WAVEGUIDE.

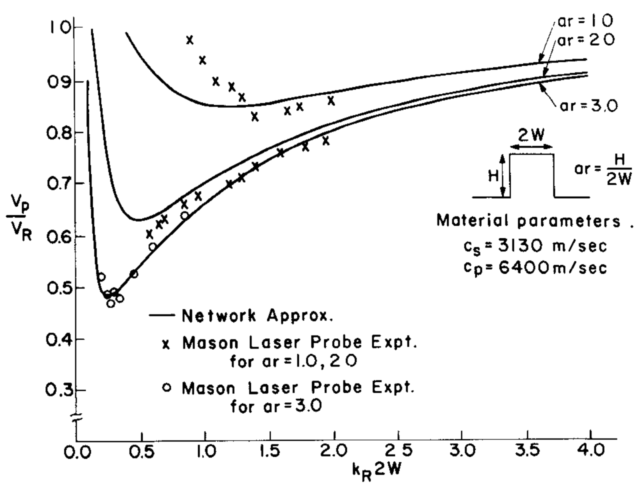
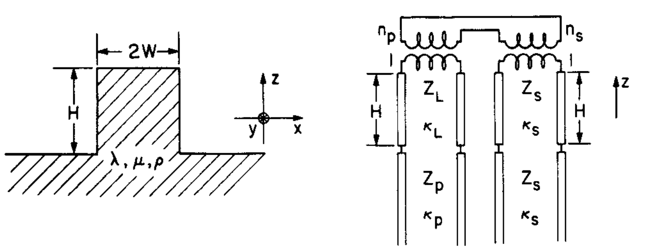


FIG. 3 - DISPERSION OF LOWEST ODD MODE (FLEXURAL MODE) OF DURALUMIN RIDGE WAVEGUIDE.



$$\kappa_s = \sqrt{k_s^2 - k_y^2} \quad , \quad Z_s = \mu \kappa_s / \omega \quad , \quad k_s = \omega \sqrt{\rho/\mu}$$

$$\kappa_p = \sqrt{k_p^2 - k_y^2} \quad , \quad Z_p = \omega \rho / \kappa_p \quad , \quad k_p = \omega \sqrt{\rho/(\lambda + 2\mu)}$$

$$\kappa_L = \sqrt{k_L^2 - k_y^2} \quad , \quad Z_L = \omega \rho / \kappa_L \quad , \quad k_L = \text{wavenumber of lowest even Lamb mode}$$

$$n_p = 1 - 2 k_y^2 / k_s^2 \quad , \quad n_s = 2 k_y / k_s$$

FIG. 4 - TRANSVERSE EQUIVALENT NETWORK FOR EVEN MODES OF RIDGE WAVEGUIDE.

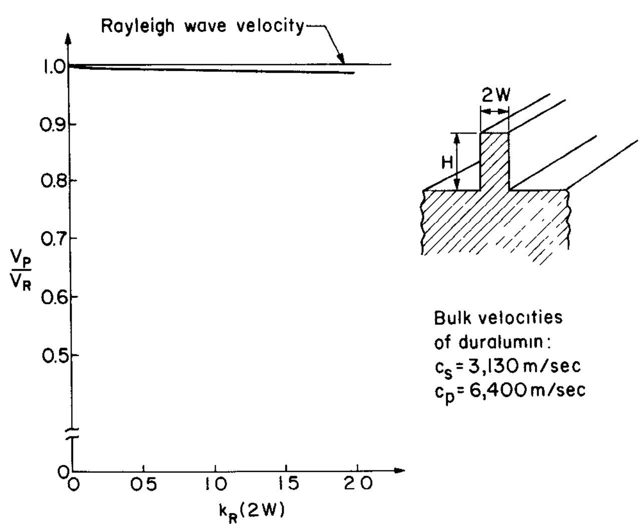


FIG. 5 - DISPERSION OF LOWEST EVEN MODE (PSEUDO-RAYLEIGH MODE) OF DURALUMIN RIDGE WAVEGUIDE; H/2W = 1, 2, 3.